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KALMAN-FILTER-BASED OBSERVER DESIGN AROUND OPTIMAL CONTROL POLICY FOR GAS PIPELINES

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SUMMARY

Seeking the optimal operating policy by an off-line controller for pipelines carrying natural gas has an inherent state estimation problem associated with deviations from demand forecast. This paper presents a Kalman-filter-based observer for the real-time estimation of deviations from the states previously obtained by an off-line controller optimally, around an expected demand function. The observer is based on the linearized form of the non-linear partial differential equations which are the state space representation of isothermal and unidirectional gas flow through a pipeline. Data for the observer are produced by a dynamic simulator. The simulator and linearized observer equations are solved using an implicit finite element method. The observer has been tested on a pipeline subject to certain deviations from demand forecast. It converges in a short span of time.

KEY WORDS: observers; gas pipelines; optimal control

1. INTRODUCTION

In recent years, demand for natural gas has been significantly increasing. This situation is acknowledged in global perspective, so that natural gas now occupies third rank in global primary energy consumption and it is widely believed that natural gas will replace oil within a few decades. This obviously brings about an increase in natural gas flow through pipelines from suppliers to consumers and the construction of new transmission and distribution systems.

When transporting natural gas, the exact conditions present in the pipeline, i.e. the distribution of pressure and flow rate, are essential information for safe and efficient operation. A method of determining pipeline conditions is to install elaborate instruments throughout the system. Since instrumentation is susceptible to drift and failure, state space estimation software is incorporated within the instrumentation. Even in gas pipeline SCADA (supervisory control and data administration) systems, real-time estimation is performed by matching real data with simulator output.[1] An observer here comes to mind as a promising mathematical tool for state space estimation.

Papers discussing and studying this estimation problem by filter-based observers have appeared infrequently in the literature. A linear observer is used by Chapman *et al.*[2] to estimate the pressure profile of a gas pipeline assuming a constant flow profile along the pipe. Lappus and Schmidt[3] present

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an approach built around a simulator. They propose a parallel simulation and observer scheme. The assumption is made that the flow rate source and off-take points are exactly known. A subsequent work[4] presents a detailed discussion of the subject. Tao and Fang[5] propose an optimization-based non-linear observer for a discretized state space model of a fluid pipeline. They utilize the method of characteristics to transform the governing equations, which are non-linear partial diifferential equations (PDEs), into ordinary differential equations (ODEs). Then these ODEs are solved numerically by an explicit finite difference method. However, as Kumar[6] stated, this approach creates instability problems if limitations on the sampling period or time interval for simulations are exceeded.

On the other hand, optimal control of gas pipelines is studied more than observer design for such systems. One of the early works in this area is Batey *et al*.[7] Wong and Larson[8] have used dynamic programming to solve limited problems such as a single compressor driving a single pipeline. A hierarchical algorithm for the control of transient flow in a large complex pipeline system is described by Larson and Wismer.[9] Osiadacz and Bell[10] have described a simplified algorithm to minimize the fuel consumption of gas engines of a gas network. In 1988, Marqués and Morari[11] presented a quadratic programming optimizer based on a dynamic simulator. Durgut and Leblebicioglu[12] tackled the control of gas pipelines under transient flow conditions by exploiting the analytical tools of variational calculus without further assumptions and simplifications other than those used to derive the governing equations.

The motivation of the present work is to investigate what should be the actual states of the gas pipeline for which the optimal operating policy is determined when the presumed demand does not materialize. The goal of this study is to design an observer for the real-time estimation of deviations from the states already evaluated by an off-line controller when some perturbations occur on the demand forecast. Therefore the system is linearized around the states evaluated by optimal control. Note that although it is also possible to design a non-linear observer, we have chosen to design a linear observer in order to reduce the computational burden, so that in an implicit way the information about optimal states and the prior information about the demand are utilized. In fact, our observer may be worthwhile when an on-line controller is to be designed to keep the system (deviating from the states calculated by an off-line controller owing to erroneous demand forecast) within desired limits. Regarding the non-linearity and time-varying characteristics of the original system, there exists no well-established controller for that type of system. However, for the linearized version the controller design literature is abundant, which is in fact one of the motivations behind the linearization methodology in this work.

In this study the mathematical model, which is the set of non-linear PDEs describing isothermal and unidirectional gas flow through a pipeline, is treated as it is. The equations are linearized and solved numerically using an implicit finite element method. This approach is a good way of obtaining a finite-dimensional system reflecting the infinite-dimensional nature of the original linearized one. A Kalman-filter-based observer is designed for these linearized equations. What the observer does is estimate deviations from the already computed states in the case of perturbations. The observation is initiated from an arbitrary deviation at the instant the simulation starts. The observer converges to real states (in fact, deviations from states evaluated by an off-line controller) in a few simulation cycles.

2. MATHEMATICAL MODEL

The mathematical model of gas flow through pipelines is described by PDEs based upon the principles of conservation of mass and momentum, the equation of state, together with a relationship accounting for the deviation of the gas from ideal gas behaviour. To construct this model, it is assumed that the flow is isothermal, unidirectional and turbulent. Model development and solution techniques for this problem have been extensively studied for more than 30 years. Most authors agree on describing the dynamics of the system by the equations

$$\frac{\partial P}{\partial t} + a\frac{\partial m}{\partial x} = 0, \tag{1}$$

$$\frac{\partial m}{\partial t} + b\frac{\partial P}{\partial x} + c\frac{m|m|}{P} = 0, \tag{2}$$

where

$$a = B^2/Ag_c, \qquad b = Ag_c, \qquad c = f/2aD.$$

Equations (1) and (2) are the equation of continuity and equation of motion for transient gas flow respectively.

To solve the equations in the time domain, it is necessary to determine the initial conditions. At the given instant t_0 (for convenience $t_0 = 0$), steady state flow is presumed. Hence the mass rate at the very beginning of time, $m(x, 0)$, is constant and known, m_0. In addition, the initial pressure distribution $P(x, 0)$ can be calculated using steady state flow relationships. Summarizing, $P(x, t)$ and $m(x, t)$ should satisfy equations (1) and (2) and the relations

$$m(x, 0) = m_0, \tag{3}$$

$$P(x, 0) = P_0(x), \tag{4}$$

subject to boundary conditions

$$m(L, t) = \alpha(t), \quad t \geqslant 0, \tag{5}$$

$$P(0, t) = \beta(t), \quad t \geqslant 0, \tag{6}$$

where $\alpha(t)$ is known and corresponds to the time-varying demand. Under these conditions one can obtain an optimal controller $\beta(t)$ which is the control policy for the inlet pressure so that the outlet pressure is as close as possible to the contract pressure and the time-varying demand is satisfied via tools of classical optimal control theory. As a result the corresponding optimal states (trajectories) $m(x, t)$ and $P(x, t)$ are found. The problem is that actual value of $\alpha(t)$ cannot be known beforehand. Thus, assuming $\alpha_a(t) \approx \alpha(t)$, we can write

$$\alpha_a(t) = \alpha(t) + \Delta\alpha(t), \tag{7}$$

$$\beta_a(t) = \beta(t) + \Delta\beta(t), \tag{8}$$

$$m_a(x, t) = m(x, t) + \Delta m(x, t), \tag{9}$$

$$P_a(x, t) = P(x, t) + \Delta P(x, t). \tag{10}$$

The linearized forms of equations (1) and (2) are

$$\frac{\partial \Delta P}{\partial t} + a\frac{\partial \Delta m}{\partial x} = 0, \tag{11}$$

$$\frac{\partial \Delta m}{\partial t} + b\frac{\partial \Delta P}{\partial x} + 2c\frac{m}{P}\operatorname{sgn}(m)\Delta m - c\frac{m|m|}{P^2}\Delta P = 0, \tag{12}$$

where m and P are as already computed in the optimal control stage. As mentioned before, this linearization is accomplished around the optimal functions P and m. Although linearization is done around optimal states, it is not necessary to restrict this linearization to those particular states.

The Δ-variables then satisfy the initial and boundary conditions

$$\Delta m(x, 0) = 0, \tag{13}$$

$$\Delta P(x, 0) = 0, \tag{14}$$

$$\Delta m(L, t) = \Delta\alpha(t), \tag{15}$$

$$\Delta P(0, t) = \Delta\beta(t). \tag{16}$$

Let us perform a finite element discretization on the linearized system:

$$\Delta m_N(x, t) = \sum_{j=1}^{N} \Delta m_j(t)\varphi_j(x), \tag{17}$$

$$\Delta P_N(x, t) = \sum_{j=1}^{N} \Delta P_j(t)\varphi_j(x), \tag{18}$$

where the functions $\varphi_j(x)$ are called shape functions and are defined on $[0, L]$ as

$$\varphi_j(x) = \begin{cases} -\dfrac{1}{\Delta x}(x - x_{j-1}), & x \in [x_j, x_{j-1}], \\ \dfrac{1}{\Delta x}(x - x_{j+1}), & x \in [x_j, x_{j+1}], \\ 0, & \text{elsewhere on } [0, L]. \end{cases} \tag{19}$$

We will utilize the weak forms[13] of equations (11) and (12):

$$\int_0^L \frac{\partial \Delta P}{\partial t}\varphi(x)\mathrm{d}x + a\int_0^L \frac{\partial \Delta m}{\partial x}\varphi(x)\mathrm{d}x = 0, \tag{20}$$

$$\int_0^L \frac{\partial \Delta m}{\partial t}\varphi(x)\mathrm{d}x + b\int_0^L \frac{\partial \Delta P}{\partial x}\varphi(x)\mathrm{d}x + 2c\int_0^L \frac{m}{P}\mathrm{sgn}(x)\Delta m\varphi(x)\mathrm{d}x - c\int_0^L \frac{m|m|}{P^2}\Delta P\varphi(x)\mathrm{d}x = 0, \tag{21}$$

where φ is an arbitrary piecewise continuous function defined on $[0, L]$.

If we perform integration by parts on (20) and (21), then

$$\int_0^L \frac{\partial \Delta P}{\partial t}\varphi(x)\mathrm{d}x + a\left(\Delta m(x, t)\varphi(x)|_0^L - \int_0^L \Delta m\frac{\mathrm{d}\varphi(x)}{\mathrm{d}x}\mathrm{d}x\right) = 0, \tag{22}$$

$$\begin{aligned} &\int_0^L \frac{\partial \Delta m}{\partial t}\varphi(x)\mathrm{d}x + b\left(\Delta P(x, t)\varphi(x)|_0^L - \int_0^L \Delta P\frac{\mathrm{d}\varphi(x)}{\mathrm{d}x}\mathrm{d}x\right) + 2c\int_0^L \frac{m}{P}\mathrm{sgn}(x)\Delta m\varphi(x)\mathrm{d}x \\ &\quad - c\int_0^L \frac{m|m|}{P^2}\Delta P\varphi(x)\mathrm{d}x = 0 \end{aligned} \tag{23}$$

Substituting ΔP_N and Δm_N in (22) and (23) yields the approximate form of the linearized system:

$$\sum_{j=1}^{N}\Delta\dot{P}_j(t)\int_0^L \varphi_j(x)\varphi(x)\mathrm{d}x + a\left(\Delta\alpha(t)\varphi(L) - \sum_{j=1}^{N} m_j(t)\varphi_j(0)\varphi(0) + \sum_{j=1}^{N} m_j(t)\int_0^L \varphi_j(x)\frac{\mathrm{d}\varphi(x)}{\mathrm{d}x}\mathrm{d}x\right) = 0, \tag{24}$$

$$\begin{aligned}&\sum_{j=1}^{N}\Delta\dot{m}_j(t)\int_0^L \varphi(x)_j\varphi(x)\mathrm{d}x + b\left(\sum_{j=1}^{N}\Delta P_j(t)\varphi_j(L)\varphi(L) - \Delta\beta(t)\varphi(0) - \sum_{j=1}^{N}\Delta P_j(t)\int_0^L \varphi_j(x)\frac{\mathrm{d}\varphi(x)}{\mathrm{d}x}\mathrm{d}x\right)\\ &\qquad + 2c\sum_{j=1}^{N}\Delta m_j(t)\int_0^L \frac{m}{P}\mathrm{sgn}(m)\varphi_j(x)\varphi(x)\mathrm{d}x - c\sum_{j=1}^{N}\Delta P_j(t)\int_0^L \frac{m|m|}{P^2}\varphi_j(x)\varphi(x)\mathrm{d}x = 0.\end{aligned} \tag{25}$$

Up to now $\varphi(x)$ was arbitrary. Now we may choose it as $\varphi_i(x)$, $i = 1, 2, \ldots, N$, in the above equations. If we introduce matrices

$$\mathbf{A} = \left[\int_0^L \varphi_j(x)\varphi_i(x)\mathrm{d}x\right]_{N\times N}, \qquad \mathbf{B} = \left[\int_0^L \varphi_j\frac{\mathrm{d}\varphi_i(x)}{\mathrm{d}x}\mathrm{d}x\right]_{N\times N}, \qquad \mathbf{C}(x) = [\varphi_j(x)\varphi_i(x)]_{N\times N},$$

$$\mathbf{D}(t) = \left[\int_0^L \frac{m}{P}\mathrm{sgn}(x)\varphi_j(x)\varphi_i(x)\mathrm{d}x\right]_{N\times N}, \qquad \mathbf{E}(t) = \left[\int_0^L \frac{m|m|}{P^2}\varphi_j(x)\varphi_i(x)\mathrm{d}x\right]_{N\times N},$$

then the system of equations (24) and (25) becomes

$$\mathbf{A}\dot{\mathbf{p}}(t) + a\{\Delta\alpha(t)[\varphi_1(L)\ \cdots\ \varphi_N(L)]^{\mathrm{T}} - \mathbf{C}(0)\mathbf{m}(t) - \mathbf{B}\mathbf{m}(t)\} = 0, \tag{26}$$

$$\mathbf{A}\dot{\mathbf{m}}(t) + b\{\mathbf{C}(L)\mathbf{p}(t) - \Delta\beta(t)[\varphi_1(0)\ \cdots\ \varphi_N(0)]^{\mathrm{T}} - \mathbf{B}\mathbf{p}(t)\} + 2c\mathbf{D}(t)\mathbf{m}(t) - c\mathbf{E}(t)\mathbf{p}(t) = \mathbf{0}, \tag{27}$$

where $\mathbf{m} = [\Delta m_1\ \cdots\ \Delta m_N]^{\mathrm{T}}$ and $\mathbf{p} = [\Delta P_1\ \cdots\ \Delta P_N]^{\mathrm{T}}$.

Further simplification is done by defining

$$\mathbf{f} = [\varphi_1(L)\ \cdots\ \varphi_N(L)]^{\mathrm{T}}, \qquad \mathbf{g} = [\varphi_1(0)\ \cdots\ \varphi_N(0)]^{\mathrm{T}},$$

which leads to

$$\mathbf{A}\dot{\mathbf{p}}(t) = a(\mathbf{B} + \mathbf{C}(0))\mathbf{m}(t) - a\mathbf{f}\Delta\alpha(t), \tag{28}$$

$$\mathbf{A}\dot{\mathbf{m}}(t) = -2c\mathbf{D}(t)\mathbf{m}(t) + (b\mathbf{B} + b\mathbf{C}(L) + c\mathbf{E}(t))\mathbf{p}(t) + b\mathbf{g}\Delta\beta(t). \tag{29}$$

These can be expressed in matrix form as

$$\begin{aligned}\begin{bmatrix}\dot{\mathbf{p}}(t)\\ \dot{\mathbf{m}}(t)\end{bmatrix} &= \begin{bmatrix}\mathbf{0} & a\mathbf{A}^{-1}(\mathbf{B} + \mathbf{C}(0))\\ -2c\mathbf{A}^{-1}\mathbf{D}(t) & \mathbf{A}^{-1}(b\mathbf{B} + b\mathbf{C}(L) + c\mathbf{E}(t))\end{bmatrix}\begin{bmatrix}\mathbf{p}(t)\\ \mathbf{m}(t)\end{bmatrix}\\ &\quad + \begin{bmatrix}-a\mathbf{A}^{-1}\mathbf{f}\\ \mathbf{0}\end{bmatrix}\Delta\alpha(t) + \begin{bmatrix}\mathbf{0}\\ b\mathbf{A}^{-1}\mathbf{g}\end{bmatrix}\Delta\beta(t).\end{aligned} \tag{30}$$

We henceforward use the following notation bringing out the comprehensive expression in the observer design stage:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \qquad \mathbf{x}(t_0) = \mathbf{x}_0, \qquad \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t), \tag{31}$$

where

$$\mathbf{A}=\begin{bmatrix}\mathbf{0} & a\mathbf{A}^{-1}(\mathbf{B}+\mathbf{C}(0))\\ -2c\mathbf{A}^{-1}\mathbf{D}(t) & \mathbf{A}^{-1}(b\mathbf{B}+b\mathbf{C}(L)+c\mathbf{E}(t))\end{bmatrix},\qquad \mathbf{B}=\begin{bmatrix}-a\mathbf{A}^{-1}\mathbf{f} & \mathbf{0}\\ \mathbf{0} & b\mathbf{A}^{-1}\mathbf{g}\end{bmatrix},$$

$$\mathbf{u}(t)=\begin{bmatrix}\Delta\alpha(t)\\ \Delta\beta(t)\end{bmatrix},\qquad \mathbf{x}(t)=\begin{bmatrix}\mathbf{p}(t)\\ \mathbf{m}(t)\end{bmatrix}.$$

According to the notation, $\mathbf{u}(t)$, $\mathbf{y}(t)$ and $\mathbf{x}(t)$ denote input sequence, output sequence and state sequence respectively, $\mathbf{x}_0$ is the initial state and $\mathbf{C}$ is the measurement matrix.

3. OBSERVER DESIGN

The observer problem consists of recursively computing an estimate $\mathbf{z}(t)$ of $\mathbf{x}(t)$ for which the error decays to zero as $t \to \infty$ while the initial condition of the observed system $\mathbf{x}_0$, is unknown; that is, to design a system

$$\dot{\mathbf{h}} = f(\mathbf{h},\mathbf{u},\mathbf{y}),\qquad \mathbf{h}(t_0)=\mathbf{h}_0 \tag{32}$$

such that

$$\lim_{t\to\infty} |\mathbf{x}(t)-\mathbf{h}(t)| = 0 \tag{33}$$

for all $\mathbf{x}_0$.

In this work we have designed a continuous time deterministic Kalman-filter- based observer for our system (equation (30)) which is the linearized form of equations (1) and (2) around a particular bias point (i.e. optimal control). Moreover, it is important to note that the linearization can be accomplished around any bias state. We applied an approach presented in Baras *et al.*'s study[14] to design an observer. In the following we will briefly describe the basic idea and methodology there.

Consider the system below recapitulating equation (31):

$$\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t),\qquad \mathbf{x}(0)=\mathbf{x}_0,\qquad \mathbf{y}(t)=\mathbf{C}\mathbf{x}(t). \tag{34}$$

Following Mortensen[15] and Hijab,[16] we associate (34) with the deterministic system

$$\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{z}(t)+\mathbf{B}\mathbf{u}(t)+\mathbf{N}\mathbf{w}(t),\qquad \mathbf{z}(0)=\mathbf{z}_0,\qquad \xi(t)=\mathbf{C}\mathbf{z}(t)+\mathbf{R}\mathbf{v}(t) \tag{35}$$

and an energy cost functional

$$J_t(\mathbf{z}_0,\mathbf{w},\mathbf{v})=\tfrac{1}{2}(\mathbf{z}_0-\boldsymbol{\mu})^{\mathrm{T}}\mathbf{Q}_0^{-1}(\mathbf{z}_0-\mu)+\tfrac{1}{2}\int_0^L(\mathbf{w}(s)^{\mathrm{T}}\mathbf{w}(s)+\mathbf{v}(s)^{\mathrm{T}}\mathbf{v}(s))\mathrm{d}s, \tag{36}$$

where $\mathbf{w}(t)\in\mathbb{R}^k$ and $\mathbf{v}(t)\in\mathbb{R}^p$ are piecewise continuous, the rank of $\mathbf{N}$ is n and $\mathbf{Q}_0$ is p.d. A minimum energy input triple ($\mathbf{z}_0^*$, $\mathbf{w}^*$, $\mathbf{v}^*$), given $\xi(s)$, $0 \leqslant s \leqslant t$, is a triple that minimizes J_t subject to (35) and produces the given output record $\xi(s)$, $0 \leqslant s \leqslant t$. The deterministic or minimum energy estimate of $\mathbf{z}(t)$, given $\xi(s)$, $0 \leqslant s \leqslant t$, is the endpoint $\hat{\mathbf{z}}(t)$ of the trajectory $\mathbf{z}^*(s)$, $0 \leqslant s \leqslant t$, of (35) corresponding to a minimum energy input triple; $\hat{\mathbf{z}}(t)=\mathbf{z}^*(t)$. It can be quite easily shown that $\hat{\mathbf{z}}$ is the solution of the Kalman filter equations

$$\dot{\hat{\mathbf{z}}}(t)=\mathbf{A}\hat{\mathbf{z}}(t)+\mathbf{B}\mathbf{u}(t)+\mathbf{Q}(t)\mathbf{C}^{\mathrm{T}}(\mathbf{R}\mathbf{R}^{\mathrm{T}})^{-1}(\xi(t)-\mathbf{C}\hat{\mathbf{z}}(t)),\qquad \hat{\mathbf{z}}(0)=\boldsymbol{\mu}, \tag{37}$$

$$\dot{\mathbf{Q}}(t)=\mathbf{A}\mathbf{Q}(t)+\mathbf{Q}(t)\mathbf{A}^{\mathrm{T}}-\mathbf{Q}(t)\mathbf{C}^{\mathrm{T}}(\mathbf{R}\mathbf{R}^{\mathrm{T}})^{-1}\mathbf{C}\mathbf{Q}(t)+\mathbf{N}\mathbf{N}^{\mathrm{T}},\qquad \mathbf{Q}(0)=\mathbf{Q}_0. \tag{38}$$

It is obvious that the deterministic estimator (37), (38) is a natural candidate for an observer for the linear systems (33) if $\xi(s) = \mathbf{y}(s)$, $0 \leqslant s \leqslant t$. Thus the Kalman filter deterministic observer for system (34) is given by

$$\dot{\mathbf{m}}(t) = \mathbf{A}\mathbf{m}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{Q}(t)\mathbf{C}^{\mathrm{T}}(\mathbf{R}\mathbf{R}^{\mathrm{T}})^{-1}(\mathbf{y}(t) - \mathbf{C}\mathbf{m}(t)), \qquad \mathbf{m}(0) = \mathbf{m}_0 = \boldsymbol{\mu}, \tag{39}$$

where $\mathbf{Q}(t)$ is as given by (37).

The design parameters are $\mathbf{Q}_0$, $\mathbf{N}$, $\mathbf{R}$ and . In our case

$$\mathbf{Q}_0 = \mathbf{I}, \qquad \mathbf{R} = \mu\mathbf{I}_2, \qquad \mathbf{N} = \eta\mathbf{I}_{2N},$$

where $\mu > 0$ and $\eta > 0$.

In Reference 16, regarding convergence of the observer, the following theorem is proved.

Theorem

The dynamical system (37), (36) is an observer for the linear control system (34) provided that ($\mathbf{C}$, $\mathbf{A}$) is detectable and $\mathbf{R}$ is p.d., rank($\mathbf{N}$) $= n =$ order of system (34). Thus $\exists$ constants $K > 0$ and $\gamma > 0$ such that

$$|\mathbf{x}(t) - \mathbf{m}(t)| \leqslant K|\mathbf{x}_0 - \mathbf{m}_0|\mathrm{e}^{-\gamma t}, \quad t > 0.$$

Note that theorem is still valid if rank($\mathbf{N}$) $< n$ but ($\mathbf{A}$, $\mathbf{N}$) is controllable.

4. RESULTS AND DISCUSSION

The developed approach has been applied to a gas pipeline. The pipeline has a length of 210^5 ft and a flow diameter of 2 ft. The friction factor is constant (0·015) throughout the pipeline. The thermodynamic data of the flowing gas are given in Table I. The system outlined above is simulated by a finite element approach. All numerical processing (simulation, observation of the system) is performed on 16 nodes (i.e. $N = 16$). The simulation or sampling interval is chosen as 5 min and is not restricted by the speed of sound, unlike the case in Tao and Fang's study (i.e. they take the sampling period as the travel time of the pressure wave across one pipeline section). The designed observer is examined for convergence on two different cases in which the demand and optimal control trajectory are calculated by an optimal control method.[12] A third case is studied to reveal the capacity of the observer not only around the optimal condition but also around any bias point.

Case A

In this simple case the system is subject to a constant demand of 300 MMscfd. The outlet pressure should be around the contact pressure of 500 psi. The best operating strategy for the inlet pressure in order to keep the outlet pressure around its given value is trivially obtained by an off-line optimal controller. It is obvious that it should be constant during operation at a value of 668 psi. However, one may expect some perturbations on the presumed demand. For the sake of simplicity a stepwise change in demand was anticipated (Figure 1(b)). This brings about deviations from the states

Table I. Thermodynamic data of flowing gas

Ambient temperature	60 °F
Gas gravity	0·55
Pseudocritical pressure	672 psia
Pseudocritical temperature	345 °R

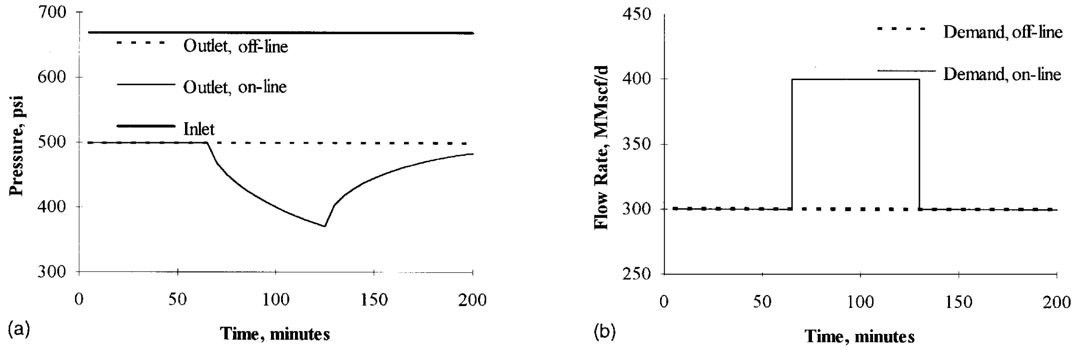


Figure 1. Case A: (a) inlet and outlet pressures of pipeline; (b) demand history at outlet

computed by the off-line controller. These deviations are estimated by the presented observer ($\mu = 1{\cdot}0$ and $\eta = 10{\cdot}0$) which has feedback from the inlet pressure and outlet flow rate.

Although an observer is a known tool whose ability is proven to obtain real states andor deviations from real states, it is prone to error. Errors of estimation are illustrated with a surface in *x*-axis (along pipe), time and error space (Figures 2(a) and 2(b). The error is defined as the difference between real states (output of simulation using real demand as boundary condition) and states via deviations estimated by the observer and states calculated by the off-line controller. Initiating the observer from an arbitrary deviation results in oscillations with a high amplitude at the beginning. However, these converge to zero fast enough in a short span of time. The error surface also reveals that the error increases at instants of change in boundary conditions and of perturbation on demand expectations. Moreover, the state estimation for the pressure is more successful than that for the flow rate.

To examine the convergence property of the designed observer under varying amplitude of perturbations on the same bias states, the demand is deliberately deviated from presumed constant flow and later this deviation is amplified by two and three times as well (cases A-1, A-2 and A-3 in Figure 3(b)). The effect of deviations on the outlet pressure is shown in Figure 3(b). Figure 4 shows how the error surface (the performance of the observer) becomes rough and starts to fluctuate with the severity of deviation. Although the error increases, it is not so significant and the peak error is about 25% of the imposed deviation on demand and about 10% of the pressure change.

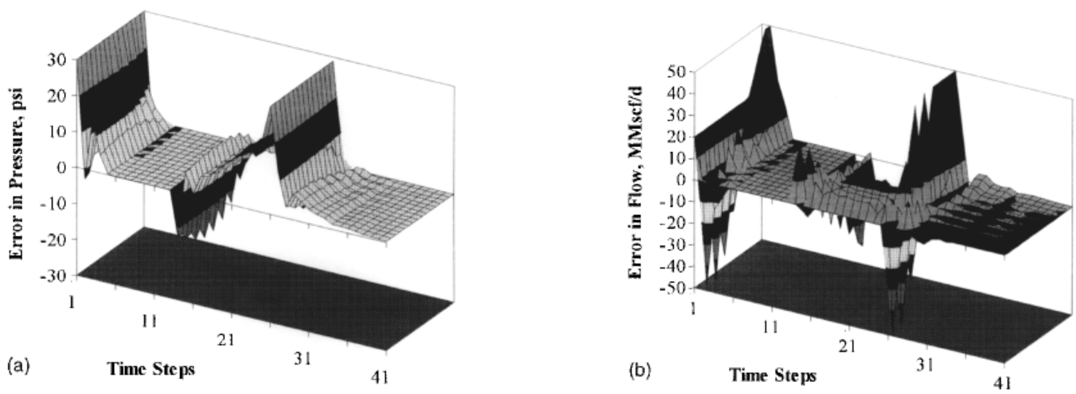
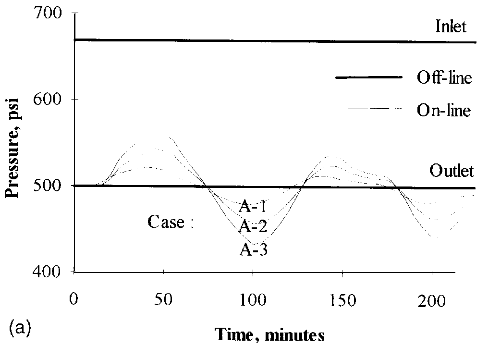


Figure 2. Case A: (a) error surface for pressure estimation; (b) error surface for flow rate estimation



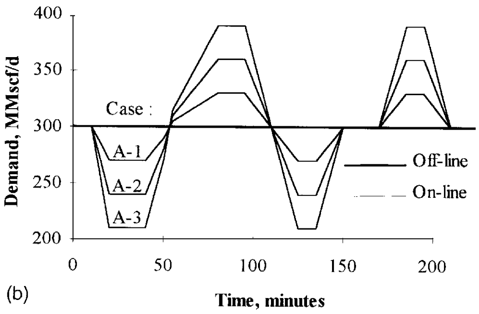


Figure 3. Case A-1–2–3: (a) inlet and outlet pressures of pipeline; (b) demand history at outlet

Case B

The second case is similar to the previous one in that the outlet pressure should again be around 500 psi. However, the demand forecast is different as shown in Figure 5(b) (bias curve). Referring to this figure, after 150 min the demand will suddenly increase by one-third; this will last for 60 min and then the flow will return to its former state. If the pipeline is controlled by an inlet pressure policy as shown by the thick line in Figure 5(a), the outlet pressure will not seriously violate its constraint. The control history is obtained by the algorithm described in Reference 12. On the other hand, the demand follows another trajectory, contrary to expectations (Figure 5(b)). In order to make the problem more realistic, the control trajectory (i.e. inlet pressure) is subjected to perturbations as shown in Figure 5(a). Therefore the states calculated by the off-line controller will differ from the real states. The deviations are estimated by the developed observer with $\mu = 0{\cdot}2$ and $\eta = 2{\cdot}0$. The error defined before is presented in Figures 6(a) and 6(b) for both state variables. This example shows that avoiding abrupt deviations smooths away the error surface after initial convergence is obtained.

Case C

As stated before, the developed observer can be applied for real-time estimation not only of deviations from the optimal state but also of deviations from any bias state. Case C investigates this proposition. The same system but with different boundary conditions (i.e. demand and pressure control history) is examined. At first the system is simulated for bias states (shown in Figures 7(a) and 7(b) as thick lines). The states obtained by adding the deviations estimated by the observer to the bias states should bring out the correspondence with the real states if the proposition is true. The results fortunately confirm the success of the observer with $\mu = 0{\cdot}2$ and $\eta = 2{\cdot}0$ (Figures 7 and 8). A comparison between real and estimated values of inlet pressure and outlet flow is presented in Figure 7; besides, Figures 8(a) and 8(b) show the error surfaces for both state variables. Since the results provide a favourable comparison, particularly considering the convergence property of the designed observer, one may state that it is a reliable tool for real-time state estimation problems.

The designed observer is tested in all three cases with arbitrary non-zero design parameters μ and η. However, they are effective with regard to the convergence rate of the observer. In order to investigate this, the last case is repeated with various pairs of values of μ and η. The mean of the absolute error surface for each pair is given in Tables II and III. In both tables the value preceding the arrow is for the whole surface, while that after the arrow is for the surface excluding the first five time steps at the beginning. Analysis of the tables indicates that there are suitable ranges for the parameters for faster convergence. However, the problem of dependence of the parameters should be noted.

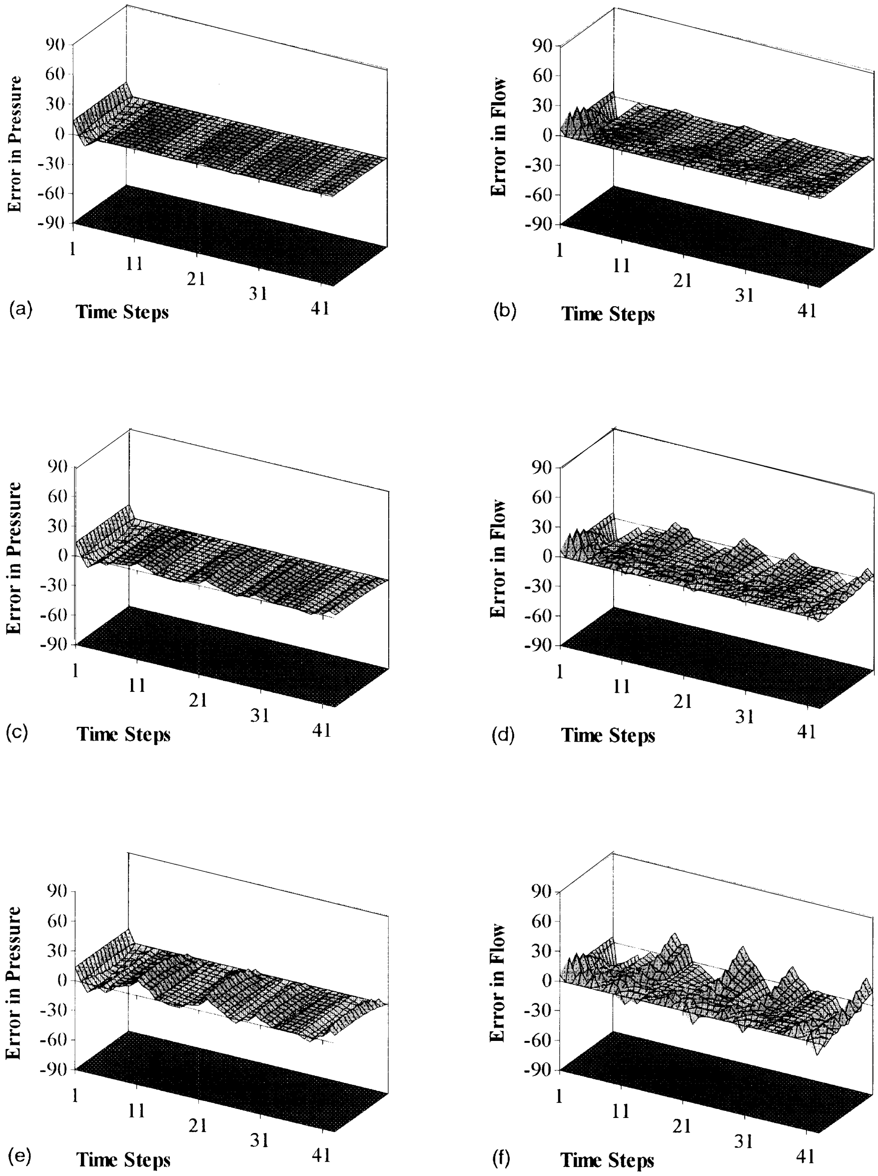


Figure 4. Error surfaces for pressure estimation in (a) Case A-1, (c) Case A-2, (e) Case A-3 and for flow rate estimation in (b) Case A-1, (d) Case A-2, (f) Case A-3

Since the on-line observation starts at an initial state different from the bias state and at an arbitrary deviation, it creates oscillations before the observer converges. Excluding this period (initial state and four simulation cycles) decreases significantly the mean of the absolute error surface.

Usually when an observer converges to the actual system, it sticks to the system and the observer error becomes zero from that time on. However, in our cases, since we use a linearized observer for a non-linear system of equations, one may observe certain jumps in the observer error even after it has converged. These jumps correspond to time instants where there are sudden changes in actual demand.

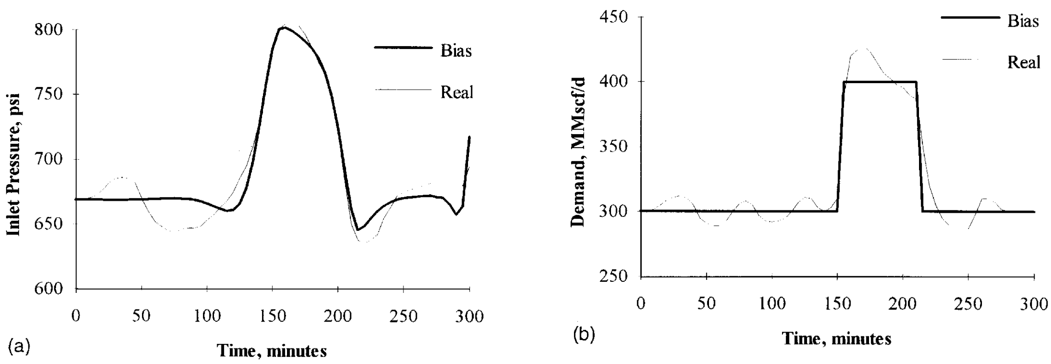


Figure 5. Case B: (a) inlet and outlet pressures of pipeline; (b) demand history at outlet

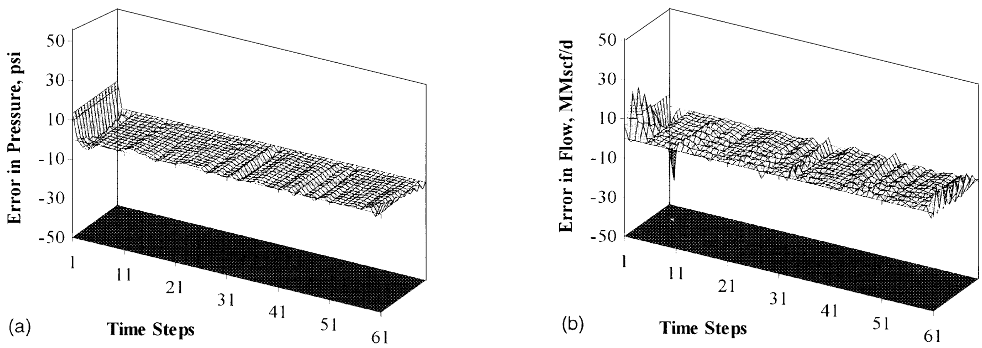


Figure 6. Case B: (a) error surface for pressure estimation; (b) error surface for flow rate estimation

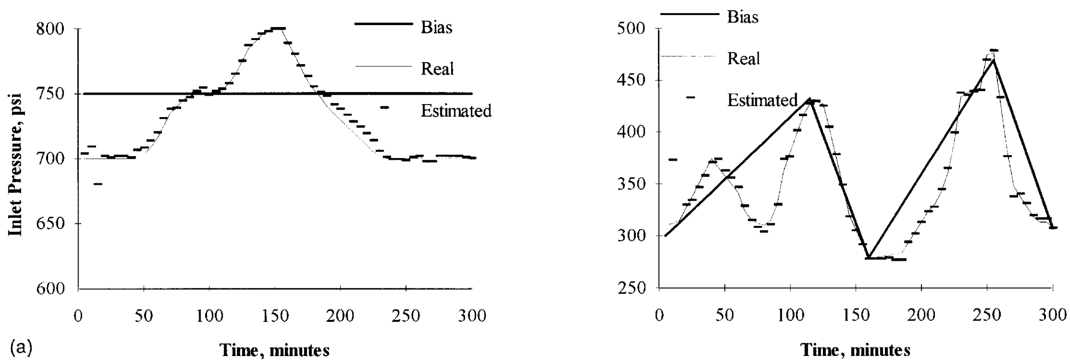


Figure 7: CASE C: (a) bias, real and estimated pressure at inlet; (b) bias, real and estimated flow rate at outlet

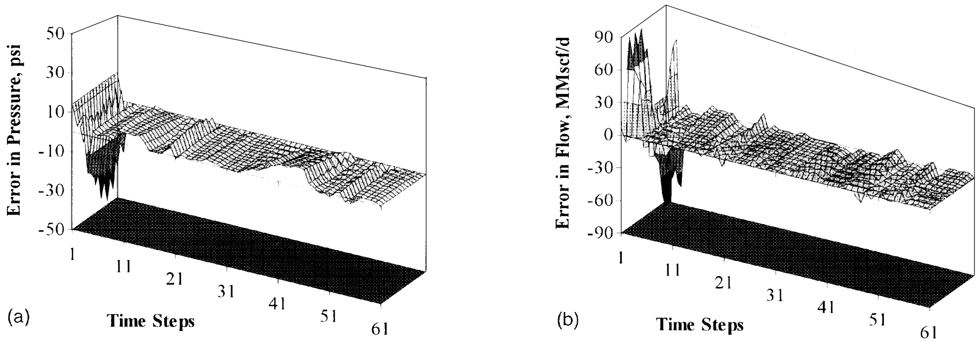


Figure 8: CASE C: (a) error surface for pressure estimation, $\mu = 0{\cdot}2$, $\eta = 2{\cdot}0$; (b) error surface for flow rate estimation, $\mu = 0{\cdot}2$, $\eta = 2{\cdot}0$

Table II. Mean of absolute error surface for different μ and η but with ratio $\eta/\mu = 10$ in Case C

	$\mu = 4$, $\eta = 40{\cdot}0$	$\mu = 1{\cdot}0$, $\eta = 10{\cdot}0$	$\mu = 0{\cdot}2$, $\eta = 2{\cdot}0$	$\mu = 0{\cdot}04$, $\eta = 0{\cdot}4$	$\mu = 0{\cdot}02$, $\eta = 0{\cdot}2$
Pressure (psi)	$5{\cdot}86 \Rightarrow 3{\cdot}71$	$3{\cdot}88 \Rightarrow 2{\cdot}22$	$2{\cdot}83 \Rightarrow 1{\cdot}93$	$3{\cdot}13 \Rightarrow 1{\cdot}90$	$11{\cdot}0 \Rightarrow 11{\cdot}6$
Flow (MMscf/d)	$8{\cdot}75 \Rightarrow 4{\cdot}59$	$5{\cdot}71 \Rightarrow 2{\cdot}43$	$4{\cdot}27 \Rightarrow 1{\cdot}87$	$5{\cdot}52 \Rightarrow 1{\cdot}74$	$15{\cdot}5 \Rightarrow 12{\cdot}9$

Table III. Mean of absolute error for different μ and η but with ratio $\eta\mu = 50$ in Case C

	$\mu = 4$, $\eta = 20{\cdot}0$	$\mu = 1{\cdot}0$, $\eta = 5{\cdot}0$	$\mu = 0{\cdot}5$, $\eta = 2{\cdot}5$	$\mu = 0{\cdot}2$, $\eta = 1{\cdot}0$	$\mu = 0{\cdot}04$, $\eta = 0{\cdot}2$
Pressure (psi)	$11{\cdot}1 \Rightarrow 9{\cdot}93$	$2{\cdot}91 \Rightarrow 2{\cdot}01$	$2{\cdot}94 \Rightarrow 2{\cdot}00$	$3{\cdot}59 \Rightarrow 2{\cdot}31$	$3{\cdot}28 \Rightarrow 1{\cdot}98$
Flow (MMscfd)	$16{\cdot}4 \Rightarrow 13{\cdot}8$	$4{\cdot}30 \Rightarrow 1{\cdot}93$	$4{\cdot}06 \Rightarrow 1{\cdot}83$	$5{\cdot}03 \Rightarrow 2{\cdot}11$	$4{\cdot}86 \Rightarrow 1{\cdot}93$

5. CONCLUSIONS

A Kalman-filter-based observer for gas pipelines has been proposed for the state estimation problem stemming from errors in demand forecast. The observer tries to estimate deviations from the states computed by an off-line controller.

The method is an alternative way of state estimation for gas flow. Although linearization is accomplished around states from optimal control, it is not necessary to restrict the observer to the estimation of states around optimal conditions. Investigations reveal that the designed observer can be utilized not only around optimal conditions but also around any bias state.

APPENDIX: NOMENCLATURE

A	cross-sectional area of pipe
a, b, c	constants
B	isothermal speed of sound in gas
D	diameter of pipe
f	friction factor
g_c	conversion factor for gravity
L	length of pipe
m	mass flow rate

P gas pressure
t time
T final time
x distance co-ordinate

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